Chapter 1

(Part 3)

Single phase AC voltage controllers

By

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Single-phase full-wave AC voltage controllers with R-L load

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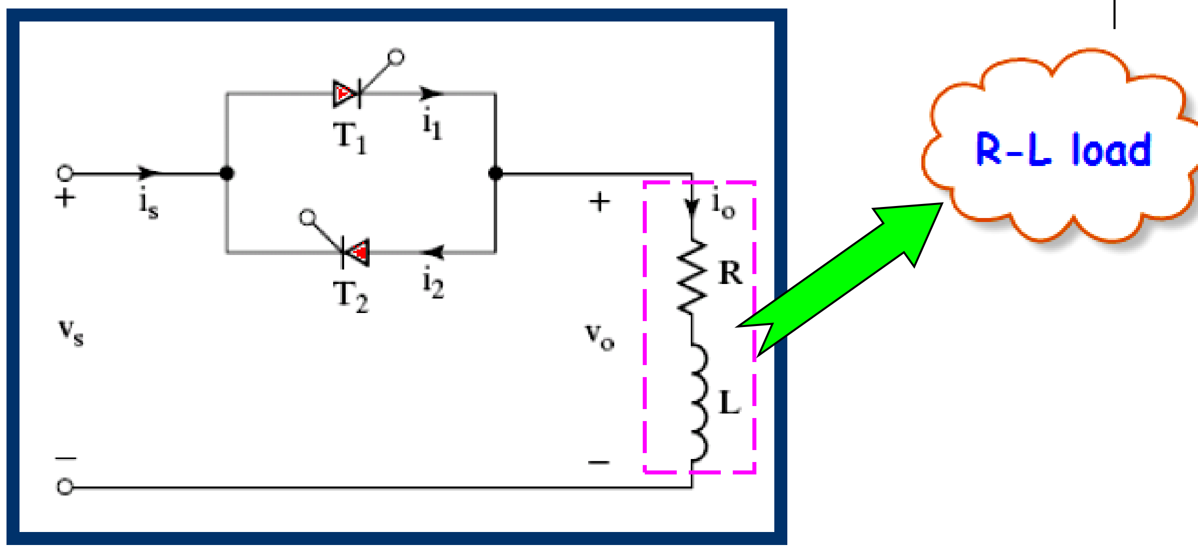
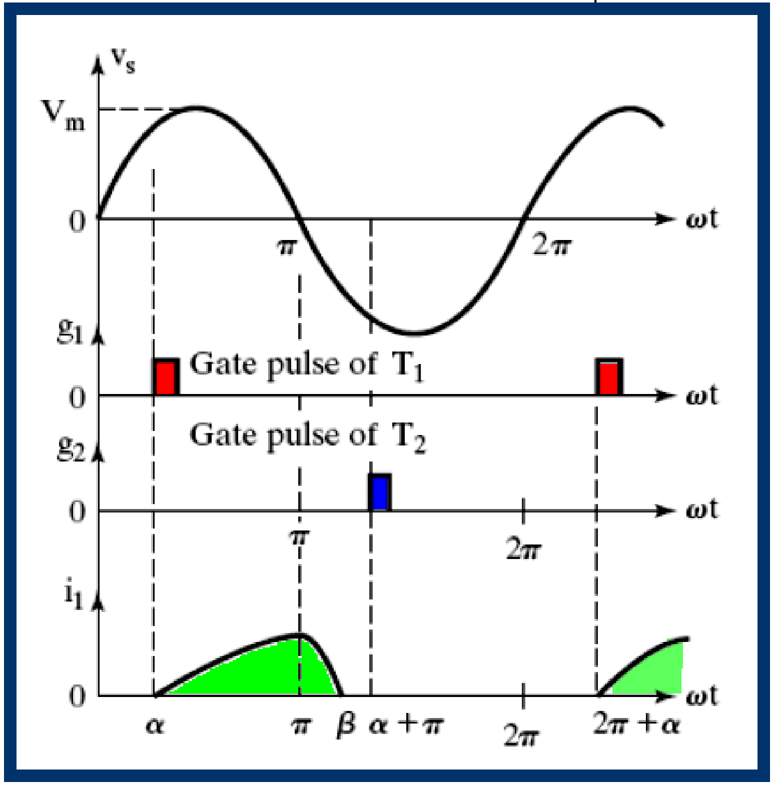


- The thyristor T_1 is forward biased during the positive half cycle of input supply and conduct when suitable pulse applied on its gate $\omega t = \alpha$.
- The thyristor T_2 is forward biased during the negative half cycle of input supply and conduct when suitable pulse applied on its gate $\omega t = \pi + \alpha$.

Single-phase full-wave AC voltage controllers with R-L load

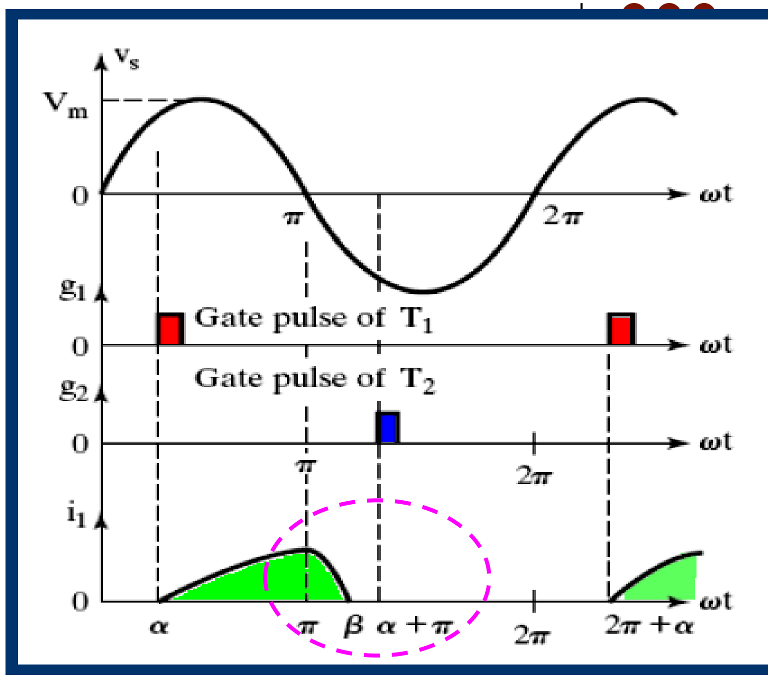
- Due to the inductance in the load, the load current i_o flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.

- The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta =$ extinction angle.



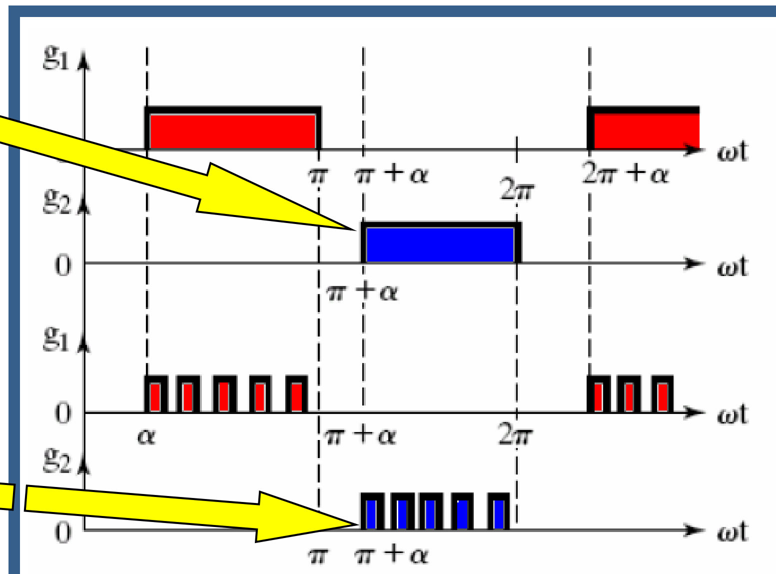
- The thyristor T_1 conducts from $\omega t = \alpha$ to β . The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle φ.

Single-phase full-wave AC voltage controllers with R-L load



In case of $\beta > \pi+\alpha$

- The problem, when (T_2) is fired at $\omega t = \pi+\alpha$, while (T_1) is still conducting due to load inductance, (T_2) cannot be turned on. This will result in asymmetric output voltage and current waveforms.
- The solution of this problem, continuous gating signals with a duration of $(\pi-\alpha)$ should be used .
- Another problem, when the current in (T_1) falls to zero, (T_2) would be turned on. But, a continuous gating signal results in high switching losses of thyristors and requires a larger isolating transformer in the gating circuit.
- In practice, a train of pulses with short durations, are normally used to overcome these problems.



Gating Signals

Waveforms of single phase full wave ac voltage controller with RL load

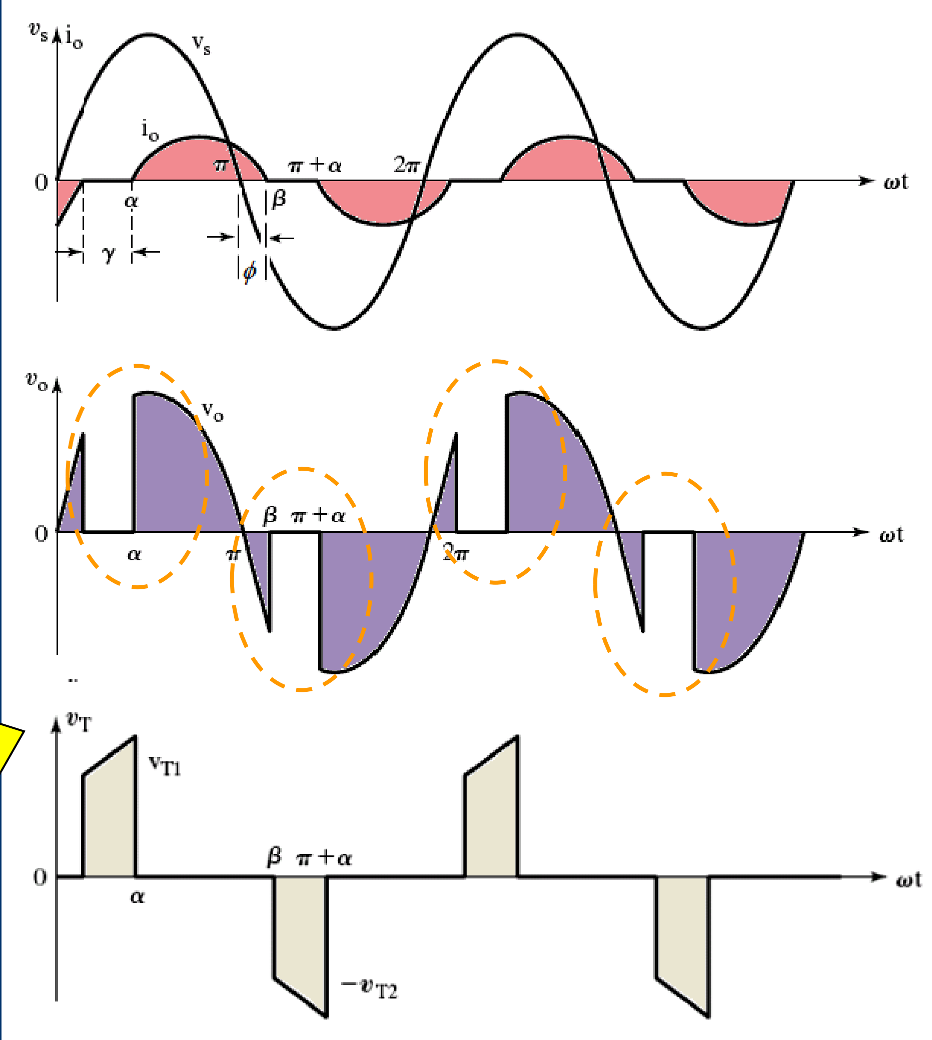
In case of ($\alpha < \varphi$)

- The load voltage and current can be sinusoidal if the firing delay angle (α) is less than the load angle (φ).

In case of ($\alpha > \varphi$)

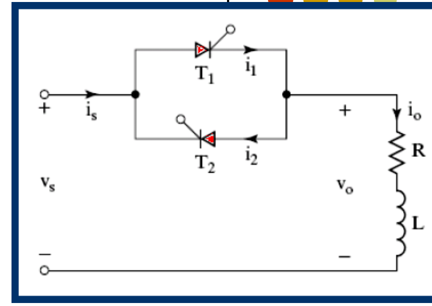
- If (α) is greater than (φ), which is usually the case, the load current would be discontinuous and non-sinusoidal.

- In discontinuous load current operation occurs for $\alpha > \varphi$ and $\beta < (\pi + \alpha)$
  i.e., $(\beta - \alpha) < \pi$, conduction angle $< \pi$.



Waveforms of single phase full wave ac voltage controller with RL load for $\alpha > \varphi$.

Analysis of Single phase full wave AC voltage controller with inductive load



The load current i_O (thyristor current i_{T1})

$v_S = V_m \sin \omega t$ ⇒ instantaneous value of the input supply voltage

- Assuming that thyristor (T_1) is triggered at $\omega t = \alpha$, The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation:

$$L\left(\frac{di_1}{dt}\right) + Ri_1 = V_m \sin \omega t$$

- The solution of the above differential equation gives the general expression for the thyristor (load) current, during this period, is in the form:

$$i_O = i_{T1} = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-t}{\tau}}$$

Where $V_m = \sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ = Load impedance. $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (power factor angle of load).

$\tau = \frac{L}{R}$ = Load circuit time constant.

Analysis of Single phase full wave AC voltage controller with inductive load

- The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for I_o equating i_o to zero and substituting $\omega t = \alpha$

$$i_O = i_{T1} = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Therefore $$A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi) \Rightarrow A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

- Substituting the value of constant A_1 from the above equation into the expression for i_o , we obtain

$$i_O = i_{T1} = \frac{V_m}{Z}\sin(\omega t - \varphi) - e^{\frac{R}{L}(\alpha/\omega - t)}\left[\frac{V_m}{Z}\sin(\alpha - \varphi)\right] \Rightarrow \text{Where } \alpha \le \omega t \le \beta$$

$$i_O = i_{T1} = \frac{\sqrt{2}V_s}{Z}\left\{\sin(\omega t - \varphi) - \sin(\alpha - \varphi)\, e^{\frac{R}{L}(\alpha/\omega - t)}\right\}$$

Analysis of Single phase full wave AC voltage controller with inductive load

Calculation of the extinction angle β

- The extinction angle β , depends upon the load inductance and is defined as the value of ω*t* at which the load current i_o falls to zero and T_1 is turned off, and it can be estimated by using the condition that $i_o = 0$, at $\omega t = \beta$

$$i_O = 0 = \frac{V_m}{Z}\left[\sin(\beta-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right]$$

As $\frac{V_m}{Z} \neq 0$ we can write

$$\left[\sin(\beta-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right] = 0$$

$$\sin(\beta-\phi) = \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

- The extinction angle β can be determined from this equation by using the iterative method of solution (trial and error method).
- In the exponential term the value of α and β should be substituted in radians.
- After β is calculated, we can determine the thyristor conduction angle δ.

Analysis of Single phase full wave AC voltage controller with inductive load

Conduction Angle δ

- After β is calculated, we can determine the thyristor conduction angle δ.

$$\delta = (\beta - \alpha)$$

- Conduction angle δ increases as α is decreased for a known value of β.

- Maximum thyristor conduction angle $\delta = (\beta - \alpha) = \pi$ radians $= 180^o$ for $\alpha \leq \varphi$.

- In case of $\alpha > \varphi$, the type of operation will be discontinuous load current, and we get $\beta < (\pi + \alpha)$, and greater than π radian or 180^o . Then, the range of extinction angle β

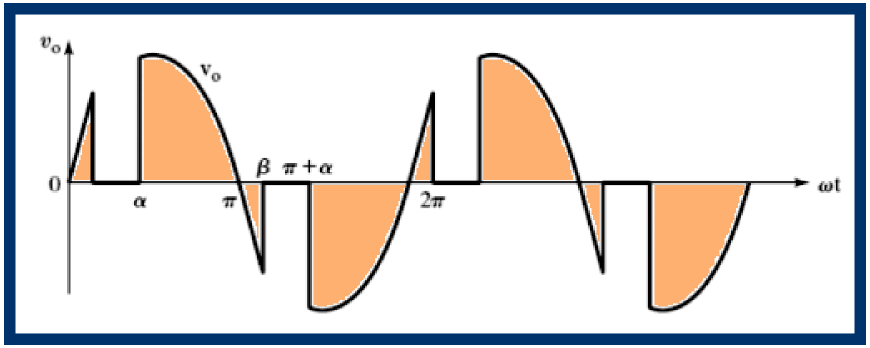
$$\pi < \beta < (\pi + \alpha)$$

Analysis of Single phase full wave AC voltage controller with inductive load

RMS Output Voltage

$$V_o = \sqrt{\frac{2}{2\pi}\int_{\alpha}^{\beta} V_m^2 \sin^2\omega t.\, d(\omega t)}$$

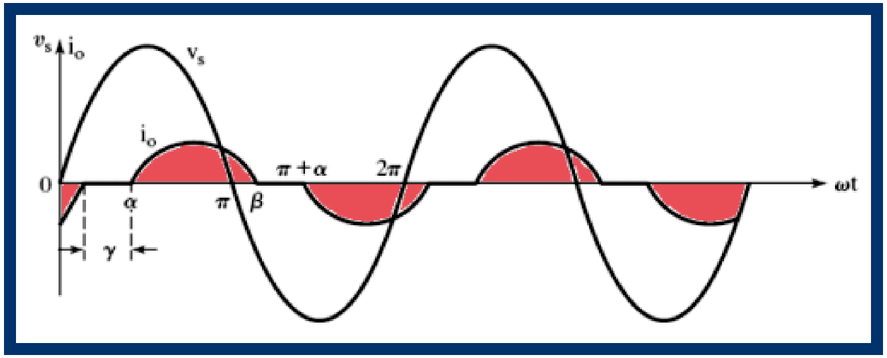
$$V_o = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\beta-\alpha)+\frac{\sin 2\alpha}{2}-\frac{\sin 2\beta}{2}\right]}$$



The Average Thyristor Current

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} i_{T_1}\, d(\omega t)\right]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\int_{\alpha}^{\beta}\left[\sin(\omega t-\varphi)-\sin(\alpha-\varphi)e^{\frac{R}{L}(\alpha/\omega - t)}\right]d(\omega t)$$



Analysis of Single phase full wave AC voltage controller with inductive load

RMS Thyristor Current $I_{T(RMS)}$

$$i_o = i_{T1} = \frac{\sqrt{2}V_s}{z}\left\{\sin(\omega t-\varphi)-\sin(\alpha-\varphi)\,e^{\frac{R}{L}(\alpha/\omega-t)}\right\}$$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi}\int_{\alpha}^{\beta} i_{T_1}^2\, d(\omega t)\right]}$$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi}\int_{\alpha}^{\beta}\left[\frac{\sqrt{2}V_s}{z}\left\{\sin(\omega t-\varphi)-\sin(\alpha-\varphi)\,e^{\frac{R}{L}(\alpha/\omega-t)}\right\}\right]^2 d(\omega t)\right]}$$

$$I_{T(RMS)} = \frac{V_s}{Z}\sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta}\left\{\sin(\omega t-\varphi)-\sin(\alpha-\varphi)\,e^{\frac{R}{L}(\alpha/\omega-t)}\right\}^2 d(\omega t)}$$

RMS Output Current

$$I_o = \sqrt{(I^2_{T(rms)} + I^2_{T(rms)})} \quad \Rightarrow \quad I_o = \sqrt{2}\; I_{T(rms)}$$

Ex .3: **A single-phase full-wave AC voltage controller supplies an RL load. The supply rms voltage is 120 V, 60 Hz. The load has R=2.5W and L=6.5 mH, the firing delay angles of thyristors are equal: $\alpha_1 = \alpha_2 = \pi/2$.**

Determine:

(a) the conduction angle of the thyristor T_1

(b) the rms output voltage

(c) the rms thyristor current

(d) the average thyristor current

(e) the rms output current

(f) the input power.

Solution

$V_S = 120$ V $\quad f = 60$ Hz $\quad R = 2.5\ \Omega \quad L = 6.5$ mH $\quad \alpha_1 = \alpha_2 = 90°$

$\varphi = \tan^{-1}(\omega L/R) = \tan^{-1}(2\pi x60x6.5x10^{-3}/2.5) = 44.4°$

A conduction angle of the thyristor T_1

$$\delta = (\beta - \alpha)$$

- In order to determine the conduction angle δ, firstly, we must find the extinction angle β from the given equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

$$\sin(\beta-\phi)=\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

- By solving this equation numerically using trial and error method , and because $\alpha > \varphi$, the extinction angle (β) can be determined with the given range

$$\pi < \beta < (\pi+\alpha) \Rightarrow \beta \cong 220°$$

$$\text{Then,}\quad \delta = \beta - \alpha = 220° - 90° = 130°$$

B rms output voltage

$$V_o = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\beta-\alpha)+\frac{\sin 2\alpha}{2}-\frac{\sin 2\beta}{2}\right]}$$

$$V_o = V_s\left[\frac{1}{\pi}\left\{(\beta-\alpha)+\frac{\sin 2\alpha}{2}-\frac{\sin 2\beta}{2}\right\}\right]^{\frac{1}{2}}$$

$$V_o = 120\left[\frac{1}{\pi}\left\{(220-90)+\frac{\sin 2x90}{2}-\frac{\sin 2x220}{2}\right\}\right]^{\frac{1}{2}} = 90.25v$$

C rms thyristor current

$$I_{T(RMS)} = \frac{V_s}{Z}\sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta}\left\{\sin(\omega t-\varphi) - \sin(\alpha-\varphi)\, e^{\frac{R}{L}(\alpha/\omega - t)}\right\}^2 d(\omega t)}$$

D average thyristor current

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\int_{\alpha}^{\beta}\left[\sin(\omega t-\varphi) - \sin(\alpha-\varphi)\, e^{\frac{R}{L}(\alpha/\omega - t)}\right] d(\omega t)$$

E rms output current

$$I_o = \sqrt{2}\; I_{T(rms)}$$

F input power factor

$$PF = \frac{P_o}{VA}$$

$$P_o = I_o^2 \times R \qquad VA = V_s \times I_s$$

Harmonic analysis of output voltage and current with RL load

rms output voltage

$$V_o = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\beta-\alpha)+\frac{\sin 2\alpha}{2}-\frac{\sin 2\beta}{2}\right]}$$

rms thyristor current

$$I_{T(RMS)} = \frac{V_s}{Z}\sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta}\left\{\sin(\omega t-\varphi)-\sin(\alpha-\varphi)e^{\frac{R}{L}(\alpha/\omega-t)}\right\}^2 d(\omega t)}$$

rms output current

$$I_o = \sqrt{2}\ I_{T(rms)}$$

Harmonic analysis of output voltage and current with RL load

The fundamental component of load voltage

$$a_1 = \frac{V_m}{2\pi}[\cos 2\alpha - \cos 2\beta]$$

$$\vartheta_1 = \tan^{-1}\frac{a_1}{b_1}$$

$$b_1 = \frac{V_m}{2\pi}[2(\beta - \alpha) + \sin 2\alpha - \sin 2\beta]$$

The nth component of load voltage

$$a_n = \frac{V_m}{\pi}\left\{\frac{\cos(1+n)\alpha - \cos(1+n)\beta}{(1+n)} + \frac{\cos(1-n)\alpha - \cos(1-n)\beta}{(1-n)}\right\}$$

$$\vartheta_n = \tan^{-1}\frac{a_n}{b_n}$$

$$b_n = \frac{V_m}{\pi}\left\{\frac{\sin(1-n)\beta - \sin(1-n)\alpha}{(1-n)} - \frac{\sin(1+n)\beta - \sin(1+n)\alpha}{(1+n)}\right\}$$

n = 3, 5, 7, ………,

Harmonic analysis of output voltage and current with RL load

The RMS of load voltage from Fourier series

n = 1, 3, 5, ….,

$$V_o = \sqrt{V_1^2 + V_3^2 + \cdots + V_n^2}$$

n = 3, 5, 7, …,

$$V_{on} = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}$$

$$V_{on} = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}$$

$$a_1 = \frac{V_m}{2\pi}[\cos 2\alpha - \cos 2\beta]$$

$$b_1 = \frac{V_m}{2\pi}[2(\beta - \alpha) + \sin 2\alpha - \sin 2\beta]$$

$$\vartheta_1 = \tan^{-1}\frac{a_1}{b_1}$$

$$a_n = \frac{V_m}{\pi}\left\{\frac{\cos(1+n)\alpha - \cos(1+n)\beta}{(1+n)} + \frac{\cos(1-n)\alpha - \cos(1-n)\beta}{(1-n)}\right\}$$

$$\vartheta_n = \tan^{-1}\frac{a_n}{b_n}$$

$$b_n = \frac{V_m}{\pi}\left\{\frac{\sin(1-n)\beta - \sin(1-n)\alpha}{(1-n)} - \frac{\sin(1+n)\beta - \sin(1+n)\alpha}{(1+n)}\right\}$$

Harmonic analysis of output voltage and current with RL load

The RMS of the load current obtained from Fourier series

$$I_o = \sqrt{I_1^2 + I_3^2 + \cdots . + I_n^2}$$

$$I_1 = \frac{V_1}{Z_1} \qquad I_3 = \frac{V_3}{Z_3} \qquad I_n = \frac{V_n}{Z_n}$$

$$Z_1 = \sqrt{R^2 + (\omega L)^2} \qquad Z_3 = \sqrt{R^2 + (3\omega L)^2} \qquad Z_n = \sqrt{R^2 + (n\omega L)^2}$$

$$\varphi_1 = \tan^{-1}\frac{\omega L}{R} \qquad \varphi_3 = \tan^{-1}\frac{3\omega L}{R} \qquad \varphi_n = \tan^{-1}\frac{n\omega L}{R}$$

Harmonic analysis of output voltage and current with RL load

Active power

$$P = V_1 I_1 \cos(\vartheta_1 - \varphi_1) + V_3 I_3 \cos(\vartheta_3 - \varphi_3) + \cdots . + V_n I_n \cos(\vartheta_n - \varphi_n)$$

$$P = I_0^2 \, R$$

$$P = \left(I_1^2 + I_3^2 + \cdots . + I_n^2\right) R$$

Reactive power

$$Q_1 = V_1 I_1 \, \sin(\vartheta_1 - \varphi_1)$$